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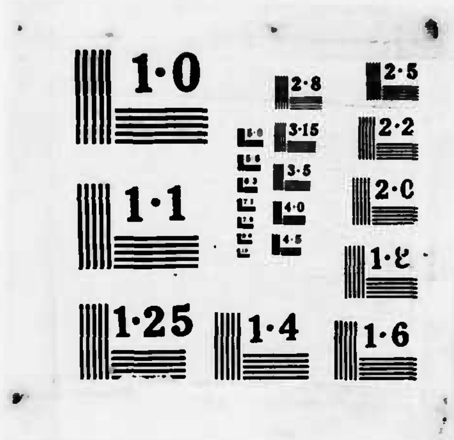
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This paper treats the robust detection problem for the case where the noise mixture model is not strongly unimodal; that is, has heavier than exponential tails. Under a constraint on the support of the contaminating density, an optimal detector is derived. Its performance is investigated using both computer-generated noises and noise records from underwater noise fields.

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Robust Detectors for Very Heavy-Tailed Noise

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Abstract

This paper treats the robust detection problem for the case where the noise mixture model is not strongly unimodal; that is, has heavier than exponential tails. Under a constraint on the support of the contaminating density, an optimal detector is derived. Its performance is investigated using both computer-generated noises and noise records from underwater noise fields.

1 - Introduction

Robust detection and estimation techniques have been developed for environments in which there is some uncertainty as to the exact distribution of the noise processes present[1,2,3]. In [4,5] the problem of detecting a signal in contaminated noise is discussed and the robust detector in terms of risk is found to be a censored probability likelihood test. The robust location estimator and small signal detector for nominal noise distributions having lighter than exponential tails (corresponding to strongly unimodal densities) are described in [3] and [6], respectively.

Huber in [3] developed the basic theory for robust estimation of location for convex classes of distribution functions. The motivation for this work was the the desire to find a technique which diminished the inordinate influence of outliers on estimates. He found that this could be done by using the system which would give the MLE for location if the noise had the distribution with the smallest Fisher's Information in the given convex class of distributions. The discussion in §2 lifts his restriction on the distributions; it contains an investigation of the analogous robust detector and location estimator for the case where the nominal has heavier than exponential tails. The paper concludes with the results of some simulations which were performed with the robust detector developed in §2.

We will discuss robust detection of a small signal in independent identically distributed mixture noise. The distribution $\mu(\mathbf{x})$ of the noise is given by

$$\mu(\mathbf{x}) = \prod_{i=1}^{n} F(x_i) = \prod_{i=1}^{n} [(1-\epsilon)G(x_i) + \epsilon H(x_i)], \qquad 0<\epsilon<1 \tag{1.1}$$

where G is the known nominal distribution, H is the unknown contaminating distribution and F is the actual distribution of the individual observations. The univariate mixture model is given by

$$F = (1-\epsilon)G + \epsilon H \quad , \quad 0<\epsilon<1 \tag{1.2}$$

where G is the known nominal distribution and H is the unknown contaminant. Furthermore, if G and H have densities g and h, respectively, we can write

$$f = (1-\epsilon)g + \epsilon h \quad , \quad 0<\epsilon<1 \tag{1.3}$$

where f is the density of F.

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2 - Robust Estimation and Detection for Noise with a Non-Strongly Unimodal Nominal Density

In [3,4] the least favorable distribution was developed for the mixture class of distributions which have a strongly unimodal nominal density. This type of density is limited to those which have lighter than exponential tails. However, many of the noise environments commonly encountered in atmospheric and oceanographic applications have been found [7,9,10] to have distributions which are much heavier tailed than exponential. It therefore seems proper to investigate the robust problem for very heavy tailed distributions.

We will find the robust small-signal detector and M-estimator for the mixture class of distributions having what will be called non-strongly unimodal nominal densities. This class is defined as follows.

Definition 1. *A unimodal density g is called non-strongly unimodal if it satisfies the following conditions:*

i) $sgn\left[\frac{-g'}{g}(x-c)\right] = sgn\,[x-c]$ *some real* c.

ii) There exist P_1 *and* P_2 *with* $P_2 < c < P_1$ *such that*

a) For $x < P_2$ *and* $x > P_1$*, then* $-g'(x)/g(x)$ *is monotonically decreasing.*

b) For $P_2 \le x \le P_1$*, then* $-g'(x)/g(x)$ *is monotonically increasing.*

iii) $\lim_{|x|\to\infty} g'(x)/g(x) = 0$.

For simplicity we will assume throughout this section that $c=0$. Note that if g is symmetric and $c=0$ then $P_2 = -P_1$.

Cauchy noise is an example of a distribution having a non-strongly unimodal density. Unfortunately, the moments of the Cauchy distribution are undefined. However, during many practical applications, the extreme tails would be truncated by the signal processor so that we can, if it seems appropriate, model a noise process as Cauchy within some finite interval of the observation space. Over such an interval all moments exist and are finite.

The members of the Johnson family [7,9,11] of distributions also have non-strongly unimodal distributions. Willett [9] found that a member of this class closely modeled the noise produced by shrimp along the ocean floor. Johnson noise is generated by the following memoryless transformation on Gaussian noise:

$$x = \lambda \sinh\left(\frac{z}{\delta}\right)$$

Here z is an observation from a unit normal process and x is an observation from the generated Johnson process. The parameter δ controls the tail weight; very large δ produces a nearly Gaussian process; and λ is a scale parameter given by

$$\lambda = \left[\frac{2\sigma^2}{e^{\frac{2}{\delta^2}} - 1}\right]^{1/2}$$

where σ^2 is the variance of the Johnson process. Because δ, and hence the tail mass, is allowed to vary continuously, the Johnson family can afford a good basis for approximating distributions with very heavy tails.

We will now investigate the properties of the absolutely continuous mixture density function f

$$f = (1-\epsilon)g + \epsilon h \tag{2.1}$$

where g is a non-strongly unimodal density and h is any density function for which the resulting f is absolutely continuous. We will also assume in this section that f is finite, unimodal and attains its maximum at $x=0$. This assumption is reasonable since many distributions encountered in nature are unimodal and we place the mode at zero without loss of generality. Hence, we have added the constraint that $\epsilon h'(x)$ is less(greater) than $-(1-\epsilon)g'(x)$ for

x greater(less) than zero.

Fact 1. *The density f is not strongly unimodal.*

Proof. From the definition of non-strongly unimodal we have

$$\lim_{|x|\to\infty} \frac{-g'}{g} = 0.$$

Then for any $\delta>0$ there exists finite $M>0$ such that $-g'(x)/g(x) < \delta$ for all $x>M$. Pick $z_0>M$. Since both f and g are finite we have $-\log f(z_0) + \log g(z_0) = r < \infty$. Furthermore, by the positivity of h, $-\log f(x) < -\log\, g(x)+c$, for all x, where $c = -\log(1-\epsilon)$. Assume f is strongly unimodal; that is, $-\log f$ is convex and, for positive x, dominates some line with constant positive slope k. Pick any δ between zero and k. Then for $z_1>z_0$

$$-\log f(z_1) + \log f(z_0) \geq k(z_1-z_0) \tag{2.2}$$

and

$$-\log g(z_1) + \log g(z_0) < \delta(z_1-z_0). \tag{2.3}$$

Combining (2.2) and (2.3) yields

$$\begin{aligned} -\log f(z_1) + \log g(z_1) &> -\log f(z_0) + \log g(z_0) + (k-\delta)(z_1-z_0) \\ &= r + (k-\delta)(z_1-z_0). \end{aligned} \tag{2.4}$$

But the right side of (2.4) can be made as large as desired by choosing z_1; hence it can be made larger than c and the positivity condition for h will be violated which, by contradiction, completes the proof.

Fact 2. *If $-f'/f$ has a limit*† *then*

$$\lim_{|x|\to\infty} \frac{-f'}{f} = 0 \tag{2.5}$$

The proof of Fact 2 is virtually identical to that of Fact 1.

In light of Facts 1 and 2 we see that we cannot contaminate a non-strongly unimodal density and produce one which is strongly unimodal. Furthermore, if f is not strongly unimodal then the derivative of $-f'/f$ must be negative for some positive arguments. In addition, since f is unimodal, $-f'(x)/f(x)$ is zero at the origin and positive for all positive values of x; therefore its derivative cannot be negative for all positive x. As a result it is not possible to fulfill the conditions of the lemma of the previous section if the nominal density of f is non-strongly unimodal.

Fact 3. *If the nonlinearity $\eta(x)$ is zero at $x=\pm\infty$ and at $x=0$ and if $sgn[\eta(x)] = sgn[x]$ then $E_F\eta' > 0$.*

Proof. Since $\eta(x)>0$ for $x>0$ and $\eta(x)<0$ for $x<0$, then $\eta'(x)$ must be positive for some interval (b,a) about the origin. Furthermore,

$$\int_{-\infty}^{\infty} \eta' = 0$$

Then, after separating the real line into $(-\infty,b]$, (b,a) and $[a,\infty)$ we have

$$0 < \int_b^a \eta' = -\int_{-\infty}^b \eta' - \int_a^{\infty} \eta'. \tag{2.6}$$

By the unimodality of f we have

$$\int_{-\infty}^{\infty} \eta' f > f(b)\left[\int_b^0 \eta' + \int_{-\infty}^b \eta'\right] + f(a)\left[\int_0^a \eta' + \int_a^{\infty} \eta'\right] = 0 \tag{2.7}$$

† For an example of a mixture density f for which $-f'/f$ has no limit see [12].

which completes the proof.

Let us now apply Fact 3. We are given two distribution functions F_1 and F_2 both of the form

$$F_i = (1-\epsilon)G + \epsilon H_i, \qquad i=1,2.$$

with density functions $f subi(x) = dF_i(x)/dx$. Let $A = \{x: \eta'(x)<0\}$ and suppose

$$\int_A h_1 = 1 \text{ and } \int_A h_2 = 0.$$

Then

$$E_{F_1}\eta' < E_{F_2}\eta' .$$

where as a consequence of Fact 3

$$|E_{F_i}\eta'| = E_{F_i}\eta' , \qquad i=1,2. \tag{2.8}$$

Hence the numerator of the expression for efficacy and the denominator of that for asymptotic variance (see [3,12]) are minimized by F_1. Similarly, if we let $B = \{x: |\eta(x)| = a \textit{ maximum}\}$ and suppose

$$\int_B h_1 = 1 \text{ and } \int_B h_2 = 0.$$

Then

$$E_{F_1}\eta^2 < E_{F_2}\eta^2. \tag{2.9}$$

Therefore, the denominator of the expression for efficacy and the numerator of that for asymptotic variance are maximized by F_1.

If $A = B$ then η must be decreasing on the set where it is at its maximum, a condition which can only be satisfied if the A and B are identical sets of discrete points. As a result, the contaminant H_1 would put all of its mass on a set of Lebesgue measure zero and would therefore be degenerate. Since we have constrained the problem so that a degenerate contaminant is not allowable we can state Fact 4 as follows.

Fact 4. *Let C be the set of all mixture distributions having finite, absolutely continuous, unimodal densities. If η satisfies the conditions given in Fact 3, then, for a non-strongly unimodal nominal density, it is not possible to maximize $E_F\eta^2$ and minimize $E_F\eta'$ with the same distribution $F \in C$.*

The underlying problem that Fact 4 formalizes is that we have no clear cut method by which to decide whether to put the contaminant mass where the nonlinearity is large in magnitude or where its derivative is negative. We will, in the remainder of this section, investigate the robust problem under the constraint that the support of the contaminant is limited to an interval. By limiting the allowable class of contaminants in this fashion we will be able to find a nonlinearity η_0 and a distribution $F_0 = (1-\epsilon)G + \epsilon H_0$ which satisfy

$$\min_{F \in C_*} E_F \eta_0' = E_{F_0}\eta_0 , \tag{2.10}$$

$$\max_{F \in C_*} E_f \eta_0^2 = E_{F_0}\eta_0^2 \tag{2.11}$$

and

$$\eta_0 = \frac{-f_0'}{f_0}$$

where C_* contains all mixtures with contaminants whose support is confined within some specified finite interval. Clearly, if Eqs. (2.10) and (2.11) are satisfied, then

$$\min_{F \in C_*} I(F) = I(F_0),$$

that is, F_0 is the minimum Fisher's Information distribution in C_*.

The robust detector and estimator for the class C_*

Let g be a non-strongly unimodal density function as in Definition 1 with $c=0$. Define the function

$$p(x) = \begin{cases} g(a_1)e^{k(x-a_1)}, & a_1 \le x < b_1 \\ g(a_2)e^{k(a_2-x)}, & a_2 \ge x > b_2 \\ g(x), & otherwise \end{cases} \tag{2.12}$$

where

$$k = \frac{-g'}{g}(a_1) = \frac{g'}{g}(a_2) \tag{2.13}$$

and $P_2 < a_2 < a_1 < P_1$ (see Definition 1).

Fact 5.

$$\left|\frac{-g'}{g}(a_i)\right| > \left|\frac{-g'}{g}(b_i)\right|. \tag{2.14}$$

Proof. We prove the fact for $i=1$; the proof for $i=2$ is virtually identical. From the definition of p, we have $p(a_1)=g(a_1)$ and $p(b_1)=g(b_1)$; hence $-log\ p(a_1) = -log\ g(a_1)$ and $-log\ p(b_1) = -log\ g(b_1)$. Therefore,

$$\int_{a_1}^{b_1} \left[\frac{-g'}{g}(x) - \frac{-g'}{g}(a_1)\right] dx = 0. \tag{2.15}$$

By the definition of g the integrand on the left side of (2.4.15) is positive for $a_1 < x < P_1$. Therefore, the integrand must be negative for some x between P_1 and b_1. But

$$\min_{x \in (P_1, b_1)} \frac{-g'}{g}(x) = \frac{-g'}{g}(b_1)$$

so the integrand must be negative at b_1, completing the proof.

Note that

$$\frac{-p'}{p}(x) = \begin{cases} k, & a_1 \le x < b_1 \\ -k, & b_2 < x < a_2 \\ \frac{-g'}{g}(x), & otherwise \end{cases} \tag{2.16}$$

which is non-decreasing on $S_1 = (b_2, b_1)$, non-increasing on $S_2 = (-\infty, b_2] \cup [b_1, \infty)$ and is constant at its maximum magnitude on $A = (b_2, a_2] \cup [a_1, b_1)$.

Fact 6 *Given two functions p_α and p_ω defined as in (2.15). If*
(1) $k_\alpha < k_\omega$ then

(2) $|a_{i_\alpha}| < |a_{i_\omega}|$ for $i=1,2$.

(3) $-log\ p_\alpha(x) \le -log\ p_\omega(x)$ for all x.

(4) $|b_{i_\alpha}| > |b_{i_\omega}|$ for $i=1,2$.

Proof. By Definition 1 and Eqs. (2.12) and (2.13)
(1) => (2),
(1) and (2) => (3),
and (3) => (4), completing the proof.

Assume, without loss of generality, that

$$\frac{-g'}{g}(P_1) \geq \frac{-g'}{g}(P_2)$$

Let a_2 be chosen as the smallest number in (P_2,c) such that

$$\frac{g'}{g}(a_2) \leq \frac{-g'}{g}(a_1) \tag{2.17}$$

Define the function

$$R(a_1) = \int_A [p - g] \quad 0<a_1<P_1 \tag{2.18}$$

where $A = (b_2,a_2] \cup [a_1,b_1)$.

Lemma. *If g and $-g'/g$ are continuous, then $R(a_1)$ is a monotonically non-increasing continuous function of a_1 with range* $(0,\infty)$.
Proof. See [12]

The main result of this section is given by the following theorem.
Theorem. *The minimum Fisher Information mixture distribution having the non-strongly unimodal nominal density g and contaminant supported on (b_2,b_1) has the density*

$$f_0 = (1-\epsilon)g + \epsilon h_0. \tag{2.19}$$

The density of of the contaminant is given by

$$\epsilon h_0(x) = \begin{cases} (1-\epsilon)[g(a_1)e^{k(a_1-x)} - g(x)] & a_1 \leq x < b_1 \\ (1-\epsilon)[g(a_2)e^{k(x-a_2)} - g(x)] & b_2 < x \leq a_2 \\ 0 & \textit{otherwise} \end{cases} \tag{2.20}$$

where

$$k = \frac{(1-\epsilon)\left\{g(a_1)[1-e^{k(a_1-b_1)}] + g(a_2)[1-e^{k(b_2-a_2)}]\right\}}{\epsilon + (1-\epsilon)\left\{\int_{a_1}^{b_1} g\,dx + \int_{b_2}^{a_2} g\,dx\right\}} \tag{2.21}$$

$$= \frac{-g'}{g}(a_1) = \frac{g'}{g}(a_2),$$

and the constants b_i, i=1,2 are found from

$$\log g(a_i) + \frac{g'}{g}(a_i)\,|x - a_i| - \log g(b_i) = 0\,; \tag{2.22}$$

$b_2<P_2<a_2<a_1<P_1<b_1$. *If g and $-g'/g$ are continuous then a minimum Fisher's Information distribution in the form of Eqs. (2.18)-(2.20) exists for all ϵ between* 0 *and* 1.
Proof. See [12]

3 - Numerical Results

In §2 we found the robust detector and estimator for the class of mixture noise distributions having a non-strongly unimodal nominal density g. Because the function $-g'/g$ was not monotonic we found it necessary to limit the support of the contaminant to an interval about the mode of the nominal density. For distributions satisfying this constraint the worst distribution in terms of Fisher's Information is given by Eqs. (2.19)-(2.21). The question which immediately comes to mind is: how does the robust detector perform if we violate the constraint on the support of the contaminant?

For the duration of this section we will confine our investigation to the robust detector; the analagous conclusions which can be drawn for the estimation problem should be apparent. In this section we will introduce some numerical results which will lend some insight into the question posed at the end of the preceding paragraph. We will look at three different but related situations. In the first, we will investigate the performance of several different detectors when the mixture noise conforms to the restriction on the support of the contaminant. Secondly, we relax this restriction so that the interval on which the contaminant is supported is a superset of the interval which was previously allowed. We then look at the performance of the same detectors as in the first case. Lastly, we investigate the performance of the robust detector for several different distributions which have support over the entire real line.

All of the figures discussed in this section plot efficacy against the natural logarithm of the mixing constant ϵ. For simplicity, we assume symmetric noise densities. The values for efficacy were computed using numerical integration.

Figs. 1-4 plot the efficacies for the optimal, robust, linear and sign detectors. The nominal density of the noise for these calculated values is given by

$$g(x) = \frac{1}{\sqrt{2\pi}}\frac{\delta}{\lambda}[1+(\frac{x}{\lambda})^2]^{-1/2}\exp\{-1/2[\delta\sinh^{-1}(\frac{x}{\lambda})]^2\} \tag{3.1}$$

which is the expression for a Johnson density with parameter δ (see the discussion of Johnson densities at the beginning of §2). For this work we used δ=1 and σ=1. The contaminant density is given by

$$h(x) = \begin{cases} \dfrac{\dfrac{1}{\sqrt{2\pi}\sigma_G}[e^{-\frac{x^2}{2\sigma_G^2}} - e^{-\frac{b^2}{2\sigma_G^2}}]}{\displaystyle\int_{-b}^{b}\frac{1}{\sqrt{2\pi}\sigma_G}\left[e^{-\frac{z^2}{2\sigma_G^2}} - e^{-\frac{b^2}{2\sigma_G^2}}\right]dz}, & -b < x < b \\ 0, & otherwise \end{cases} \tag{3.2}$$

where, due to the symmetry we have imposed, b_2 and b_1 from (2.20) have become $-b$ and b, respectively. For Figs. 1 and 3 σ_G=1 and for Figs. 2 and 4 σ_G=10.

As we know from §2, the efficacy of the robust detector cannot be any lower for any member of the class of mixture distributions satisfying the constraint on the support of the contaminant than it is for the worst distribution given by Eqs. (2.19)-(2.21). However, this tells us nothing about how *well* the detector can perform. Figs. 1 and 2 show that for the mixture density given by (3.1) and (3.2) the performance is not appreciably worse for the robust detector than it is for the detector which is optimal for those densities. Note that in Fig. 2 the performance is slightly worse than it is in Fig. 1. This appears to be due to the fact that for higher σ_G more of the contaminant mass falls within the intervals $(-b, -a]$ and $[a, b)$. Within $(-b, b)$, the nonlinearity for the robust detector has minimum slope and maximum magnitude on these two intervals. From the discussion in the previous section, it is clear that this will tend to reduce the efficacy of the detector when compared with the case wherein the contaminant mass is more concentrated around the mode of the nominal density.

Figs. 3 and 4 plot the efficacy of the robust detector for the noise given by Eqs. (3.1) and (3.2) along with that for the sign and linear detectors. From these graphs we can readily see that the performance of the robust detector is far superior to that of the others, especially that of the linear detector. Both the linear detector and the sign detector have been widely used and, at least in this case, it is apparent that better results are readily obtainable.

Allowing the support of the contaminant to exceed the the interval $(-b, b)$ by a small amount appears to have little effect on performance. In Figs. 5-8 the contaminant is supported on $(-b - 1/10, b + 1/10)$; that is, it exceeds $|b|$ by ten percent of the standard deviation of the nominal, which in this case is 0.1. There is a slight but largely insignificant change in the performance from the situation where the contaminant is confined to $(-b, b)$.

Figs. 9 and 10 compare efficacy for the robust detector in two different mixture noise environments. The first noise has the worst distribution as described in §2; the second has the same non-strongly unimodal nominal as the worst density and a contaminant which is supported on the entire real line. If, in fact, there exists a distribution which has minimum Fisher's Information for all mixtures with a given nominal then it follows that the detector which is optimal for that distribution must have its minimum efficacy for that same distribution; that is, it must satisfy the saddlepoint condition. This is the crux of the robust problem; to find a lower bound on the performance of a device for a set of possible circumstances under which the device may be operated. While we have not shown that the distribution given by the theorem of §2 has minimum Fisher's Information for the set of *all* mixture distributions having a given non-strongly unimodal nominal (we have done so for those with contaminants having bounded support on (b_2, b_1)) we can still find a lower bound for the performance for distributions having specified contaminants.

In Figs. 9 and 10 we show the efficacy for the robust detector for the worst noise distribution and for a noise distribution with contaminant supported along the entire real line. In both cases the nominal distribution is Johnson with $\delta=1$ and $\sigma=1$. The contaminant is Normal in Fig. 9 and is Johnson($\delta=1$) in Fig. 10. In both cases the variance of the contaminant is unity. We have also investigated a number of other contaminants including Cauchy, Laplace, Logistic, and Johnson with various values of δ. The results are very similar to Figs. 9 and 10. From the figures it is apparent that, for the set of contaminants tested, the worst distribution provides a lower bound on performance, as we would expect.

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REFERENCES

[1] J. W. Tukey, A survey of sampling from contaminated distributions, in *Contributions to Probability and Statistics* (ed. Olkin), pp. 448-485, Stanford University Press, 1960.

[2] F. R. Hampel, "A general qualitative definition of robustness," *Ann. Math. Statist.*, vol. 42, no. 6, pp. 1887-1896, 1971.

[3] P. J. Huber, "Robust estimation of a location parameter," *Ann. Math. Statist.*, vol. 35, pp. 73-101, Mar. 1964.

[4] ----, "A robust version of the probability ratio test," *Ann. Math. Statist.*, vol. 36, pp. 1753-1758, Dec. 1965.

[5] R. D. Martin and S. C. Schwartz, "Robust detection of a known signal in nearly Gaussian noise," *IEEE Trans. Inform. Theory*, vol. IT-17, pp. 50-56, Jan. 1971.

[6] S. A. Kassam and J. B. Thomas, "Asymptotically robust detection of a known signal in contaminated non-Gaussian noise," *IEEE Trans. Inform Theory*, vol. IT-22, pp. 22-26, Jan. 1976.

[7] E. Modugno, "The detection of signals in impulsive noise," *Ph.D. Dissertation*, Dept. of Electrical Engineering and Computer Science, Princeton University, Princeton, NJ, 1982.

[8] H. L. Royden, *Real Analysis,* Macmillan Publishing Co., Inc., pp. 157-180, New York, 1968.

[9] P. K. Willett, "The analysis of some undersea noise with applications to detection," to appear in *Proceedings of the 24th Allerton Conference on Communication, Control and Computing,* Oct. 1986.

[10] J. Veitch, "A characterization of Arctic undersea noise," *J. Acoust. Soc. Am.*, vol. 77, pp. 989-999, March 1985.

[11] N. L. Johnson and S. Kotz, *Distributions in Statistics: Continuous Univariate Distributions,* John Wiley and Sons, New York, 1971.

[12] D. Warren, "Topics in robust estimation and detection and in quasi-optimal detection," *Ph.D. Dissertation,* Department of Electrical Engineering, Princeton University, Princeton, NJ, 1986.

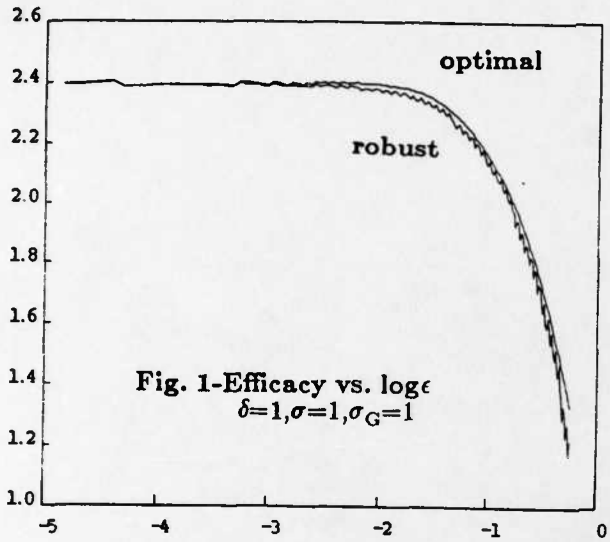

Fig. 1-Efficacy vs. logϵ
δ=1,σ=1,σ_G=1

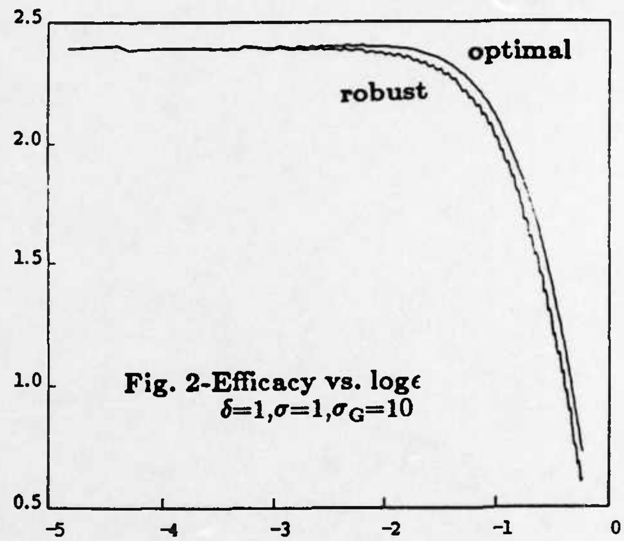

Fig. 2-Efficacy vs. logϵ
δ=1,σ=1,σ_G=10

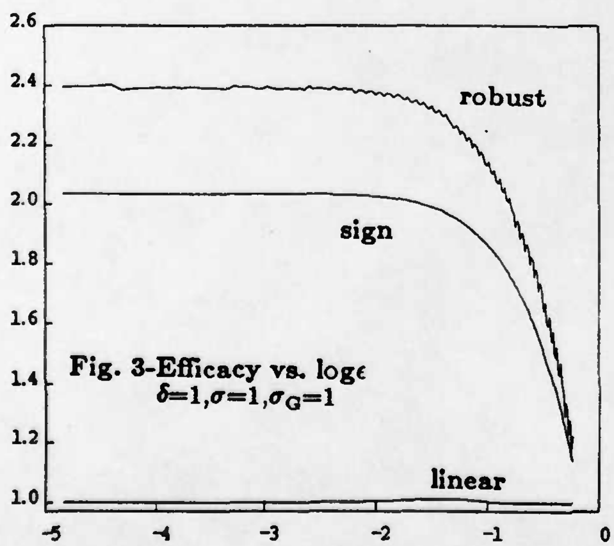

Fig. 3-Efficacy vs. logϵ
δ=1,σ=1,σ_G=1

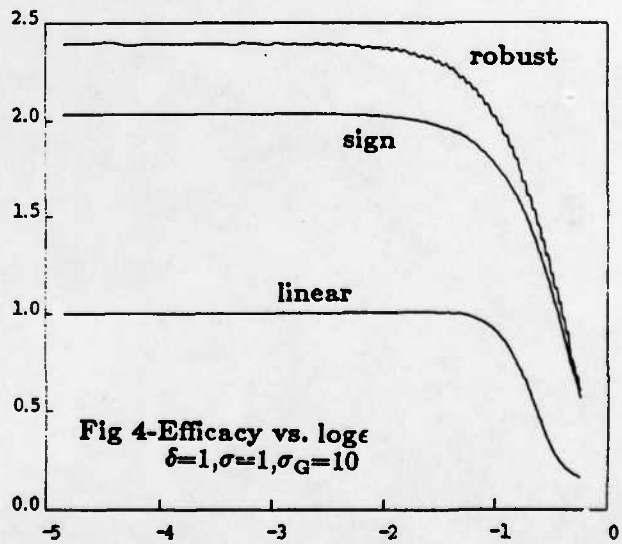

Fig 4-Efficacy vs. logϵ
δ=1,σ=1,σ_G=10

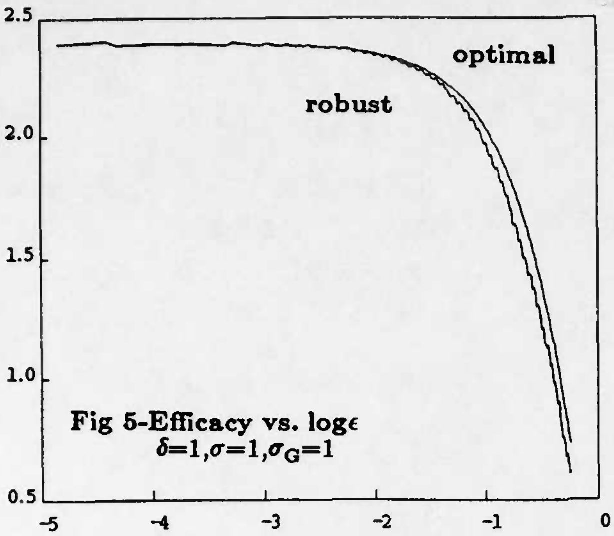

Fig 5-Efficacy vs. logϵ
δ=1,σ=1,σ_G=1

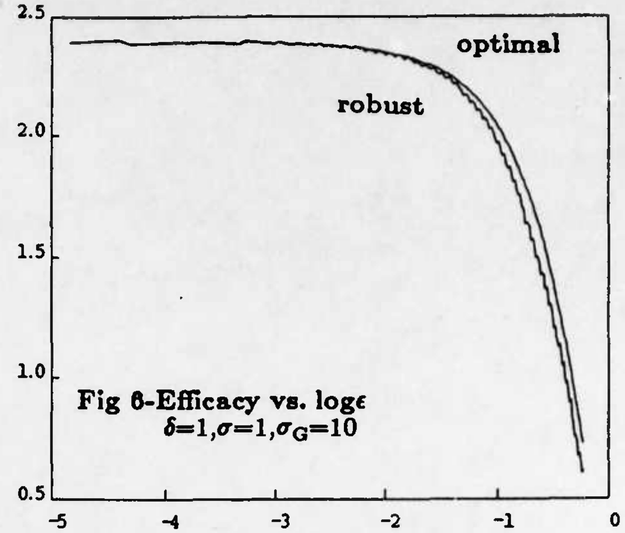

Fig 6-Efficacy vs. logϵ
δ=1,σ=1,σ_G=10

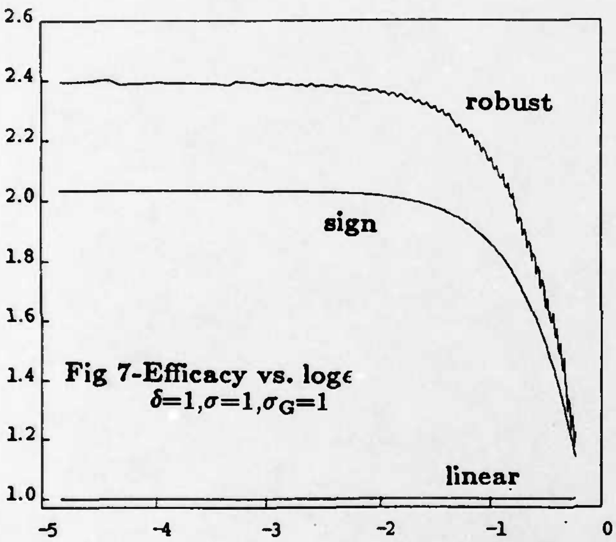

Fig 7-Efficacy vs. logϵ
δ=1,σ=1,σ_G=1

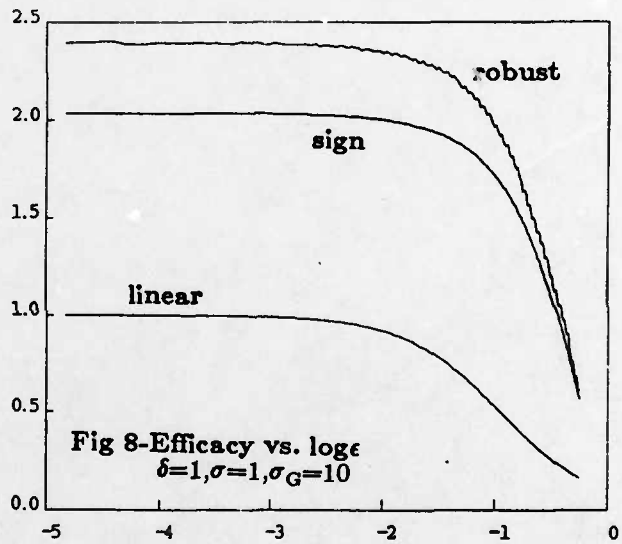

Fig 8-Efficacy vs. logϵ
δ=1,σ=1,σ_G=10

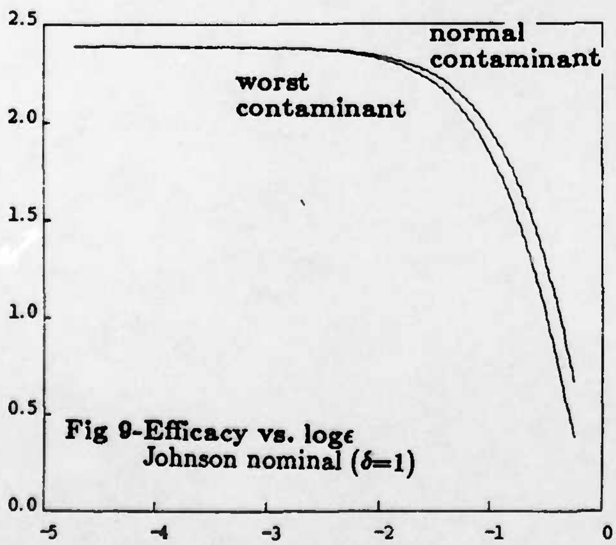

Fig 9-Efficacy vs. logϵ
Johnson nominal (δ=1)

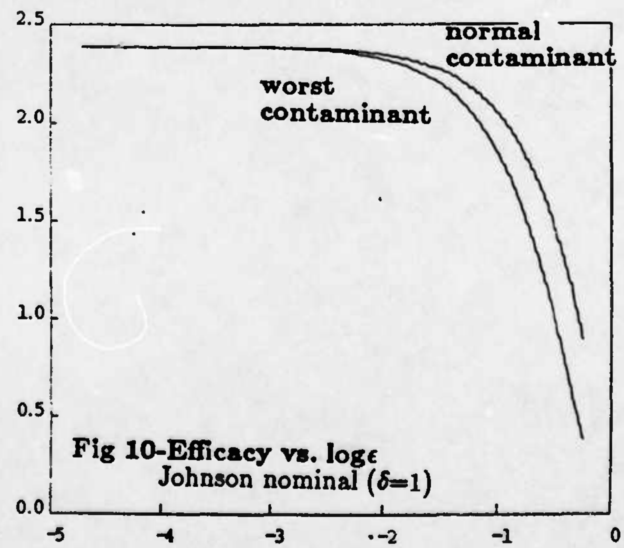

Fig 10-Efficacy vs. logϵ
Johnson nominal (δ=1)

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